EFRC Training Workshop
Basic training

Introduction to Thermodynamics and
Sizing of Reciprocating Compressors

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Content

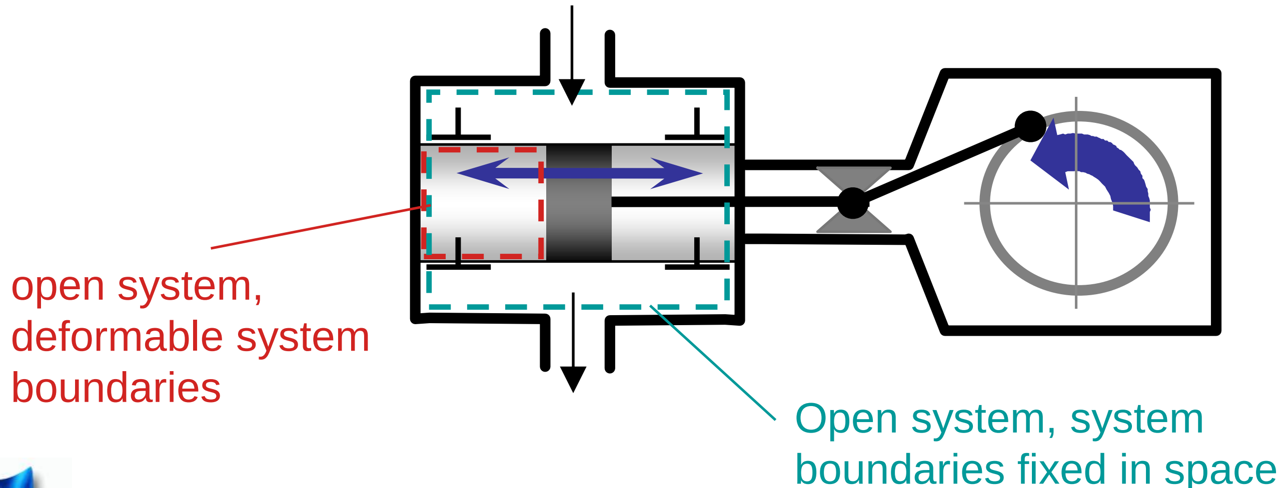
- **Basics**
- **Compression cycle**
- **Sizing of reciprocating compressors**

Basics – Thermodynamic System

Thermodynamic Systems

Separated by system boundaries (material or virtual) from surroundings (=everything except the system)

- Closed system....no transfer of matter across system boundaries
- Open system....transfer of matter across system boundaries
- Isolated system....no interaction with surroundings at all



Basics - State Variables

Thermodynamic State Variables

Macroscopic properties of a system

- **Volume *V*, Mass *m*, Energy *u*, Enthalpy *h***
- **Pressure**

– Absolute pressure = Gauge pressure + Atmospheric pressure

– Units:

1 Pa = 1 Pascal = 1 N/m²

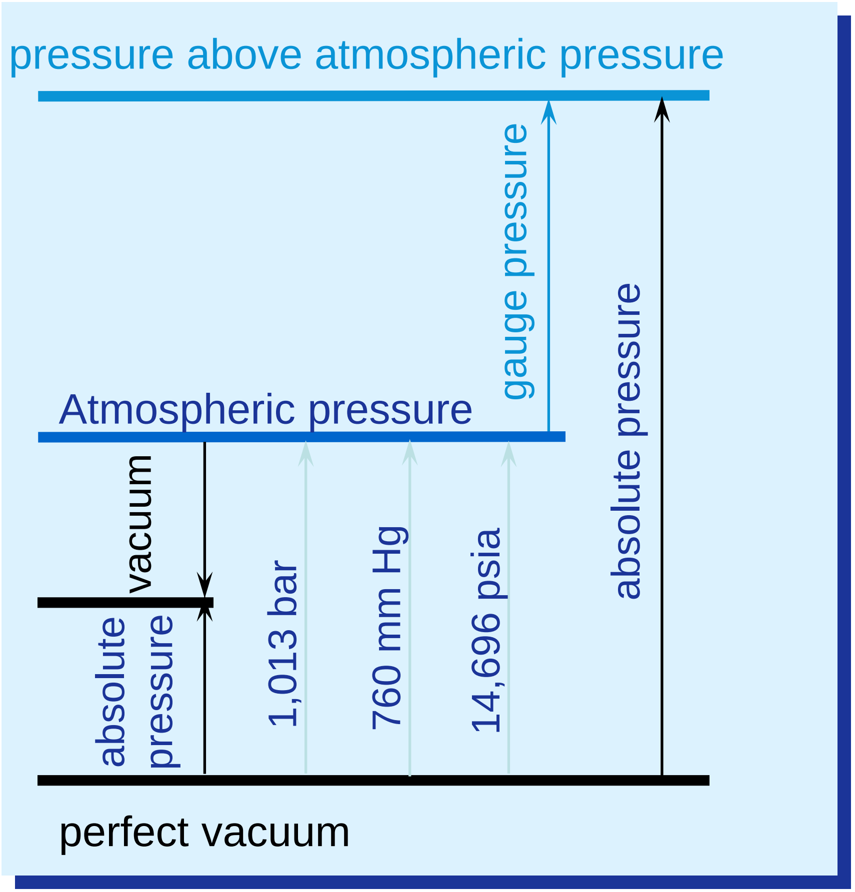
1 bar = 100.000 Pa

1 psi = 6894 Pa

– Standardard conditions:

atmospheric pressure

p_0 = 1,0133 bara



Basics - State Variables

- **Temperature**

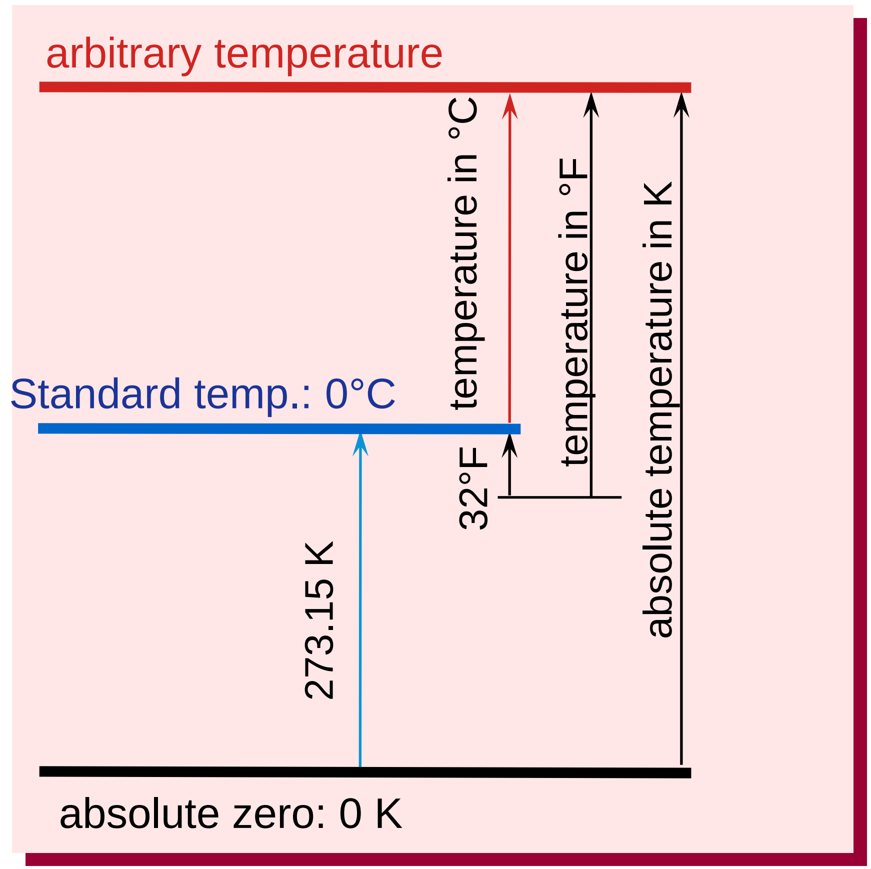
  – Absolute temperature T K
  Celsius temperature $\vartheta_{°C}$ °C
  Fahrenheit scale $\vartheta_{°F}$ °F

  – Units:

  1 K = Kelvin K
  $\vartheta_{°C}$ = T - 273.15 °C
  = ($\vartheta_{°F}$ - 32°F) / 1,8
  $\vartheta_{°F}$ = 1,8 $\vartheta_{°C}$ + 32 °F

  – Standardard conditions:

  T = 273.15 K = 0°C = 32°F



Basics - State Variables

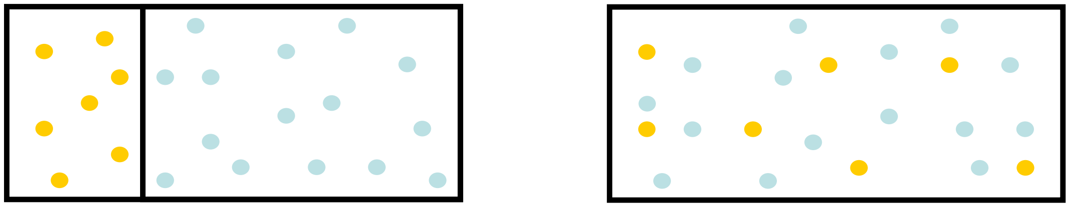
- **Molar Mass**
  - Is given by the weight of substance (chemical compound, molecules) divided by the amount of the substance
  - Unit:

    Molar Mass *M* g/mol
  - 1 Mol consists of roughly $6.022*10^{23}$ particles

Gas	Formula	Molar weight [g/mol]
Air	AIR	28.97 (see example)
Ammonia	NH_3	17.02
Hydrogen	H_2	2.02
Methane	CH_4	16.04
Ethene	C_2H_4	28.05
Nitrogen	N_2	28.01
Oxygen	O_2	32.00
Carbondioxide	CO_2	44.01

Basics - State Variables

- **Molar Mass for gas mixtures**
  - Under consideration: Mixture of n gases i=1,2,...n



  - For ideal gases Dalton's law is true
    Pressure of mixture = sum of partial pressures
  - Partial pressure pi.... pressure gas i would have if it alone occupied the volume *V* at the temperature of the mixture

$$M = \sum_{i=1}^{n} \frac{n_i}{n} * M_i$$

  - molar mass of the mixture
  - where *n* denotes the amount of substances

Basics - State Variables

- **Molar Mass for gas mixtures**

  Measures for concentration

  mole fraction $y_i := n_i / n$

  volumetric fraction $\alpha_i := V_i / \sum_{i=1}^{n} V_i$

  $\alpha_i = y_i$ for ideal gas mixtures

Example: Molar mass of dry air

Dry air consists of approx. 78% N_2, 21 %O_2 und 1 %Ar (volume fraction). What is the molar mass of the air?

$$M = 0.78 * 2 * 14 kg/kmol + 0.21 * 2 * 16 kg/kmol$$
$$+ 0.01 * 40 kg/kmol = 29.0 kg/kmol$$

Basics – Equation of State

Thermal State Function

Defines the temperature *T* of a system dependent on pressure *p* and Volume *V*

- **Ideal gas law** $\boxed{p * V = m * R * T}$
  - with $R = \mathcal{R}/M$
  - Specific gas constant R
  - Universal gas constant $\mathcal{R} = 8.314$ [J/mol/K]
  - Molar weight M
  - Mass m
- **Real gas**
  - High temperatures: ionization and chemical reactions
  - High pressures, temperatures: molecular interaction

→ Different equations of state (Compressibility Factor *Z*, Redlich-Kwong) or database of measurement data must be used

Basics – Equation of State

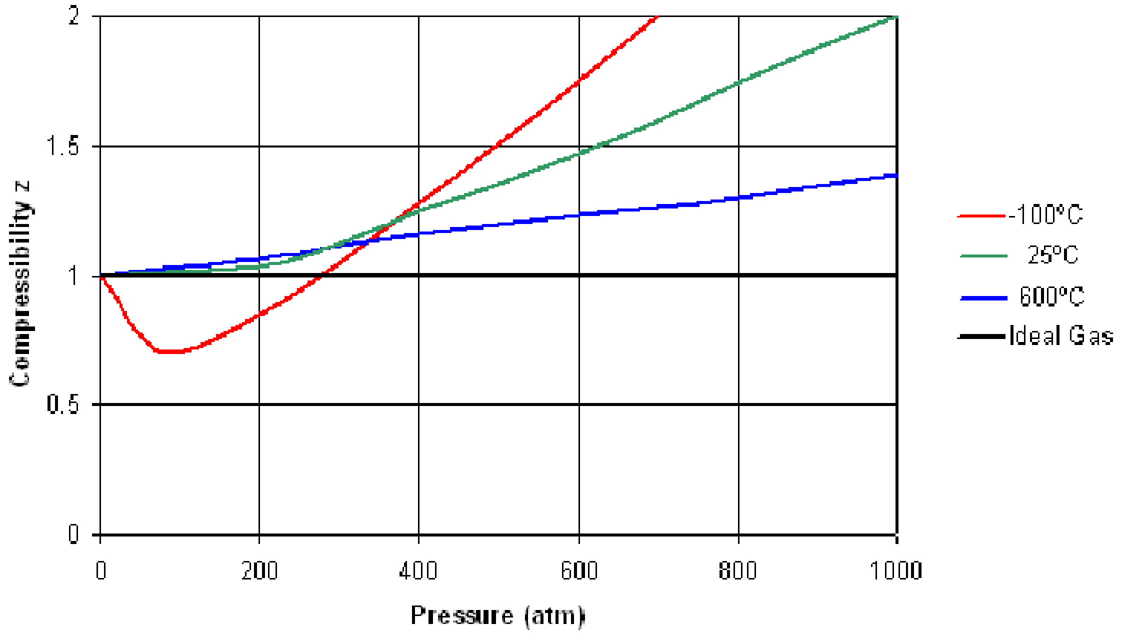
- **Real gas - Example Compressibilty factor**

  defined by

  $$p*V = m*R*T*Z$$

  – describes deviations from ideal gas law
  – *Z=Z(p,T)* has to be determined for each gas
  – Taking real gas effects into account usually requires numerical simulations

Compressibility factor of Nitrogen



Basics – Conservation Laws

1st Thermodynamic Law

Describes the conversation of energy

$$dE = d_e W + d_e Q + d_e{}^{(m)} E$$

$$\frac{dE}{dt} = \dot{W} + \dot{Q} + \dot{m} * \left[u_{in}^{(m)} + e_{kin,in}^{(m)} + e_{pot,in}^{(m)} - \left(u_{out}^{(m)} + e_{kin,out}^{(m)} + e_{pot,out}^{(m)} \right) \right]$$

Change of System Energy = performed Work + added Heat + added Energy (internal, kinetic and potential energy) of mass flow $\dot{m}$ to the system

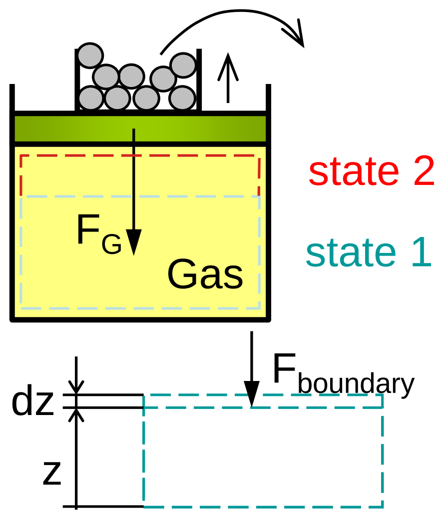
Conservation of Mass

$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

Change of Mass in the System = mass flow into the system – mass flow out of the system

Basics – Special Cases 1st Law

Example 1: closed system, quasistatic, adiabatic (without heat transfer) expansion of gas



$$dE = d_e W + d_e Q + d_e^{(m)} E \Rightarrow U_2 - U_1 = W_{12}$$

Work done on system....$d_e W = - F_{boundary} * dz$

Quasistatic change of state: pressure inside cylinder homogeneous …. $F_{boundary} = p * A$

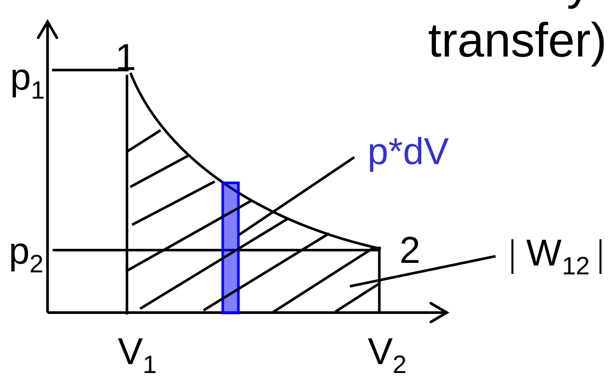
quasistatic work $d_e W = - p*A*dz = - p*dV$

$$\Rightarrow \quad U_2 - U_1 = -\int_1^2 p * dV$$

For any isentropic or adiabatic expansion (no heat transfer) change of conditions following equation is true

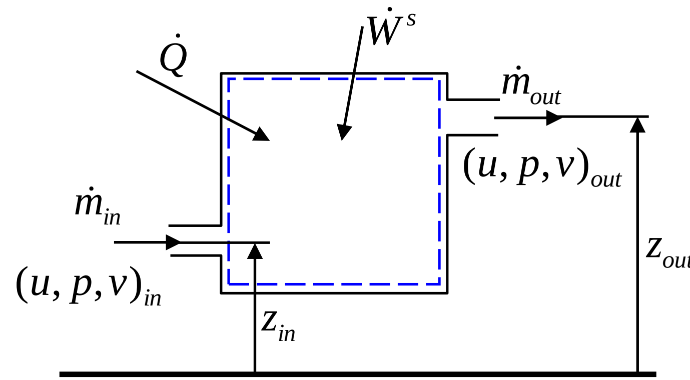
$$\boxed{p * V^{\kappa} = const}$$

with isentropic coefficient κ



Basics – Special Cases 1st Law

Example 2: open system, steady state



Mass balance:

$$\frac{dm}{dt} = 0 = \dot{m}_{in} - \dot{m}_{out} \Rightarrow \dot{m}_{in} = \dot{m}_{out} = \dot{m}$$

Energy balance:

$$\frac{dE}{dt} = 0 = \dot{W} + \dot{Q} + \dot{m} * \left[u_{in}^{(m)} + e_{kin,in}^{(m)} + e_{pot,in}^{(m)} - \left(u_{out}^{(m)} + e_{kin,out}^{(m)} + e_{pot,out}^{(m)}\right)\right]$$

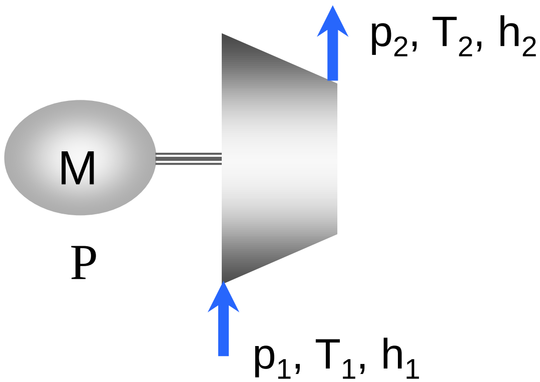
Dividing total work W into shaft work W^s and flow work $d_e W^f = p^{(m)} v^{(m)} d_e m$ (required to move fluid through system boundaries) gives

$$0 = \dot{W}^s + \dot{Q} + \dot{m} * [\underbrace{u_{in}^{(m)} + p_{in}^{(m)} * v_{in}^{(m)}}_{:= h_{in}^{(m)}} + e_{kin,in}^{(m)} + e_{pot,in}^{(m)} - (\underbrace{u_{out}^{(m)} + p_{out}^{(m)} * v_{out}^{(m)}}_{:= h_{out}^{(m)}} + e_{kin,out}^{(m)} + e_{pot,out}^{(m)})]$$

with the specific enthalpy $h := u + p * v$

Basics – Adiabatic Power Consumption

Adiabatic compression of ideal gas:
(kinetic and pot. energy not considered)



$$0 = P + \dot{m} * (h_1 - h_2) \Rightarrow P = \dot{m} * c_p * (T_2 - T_1)$$

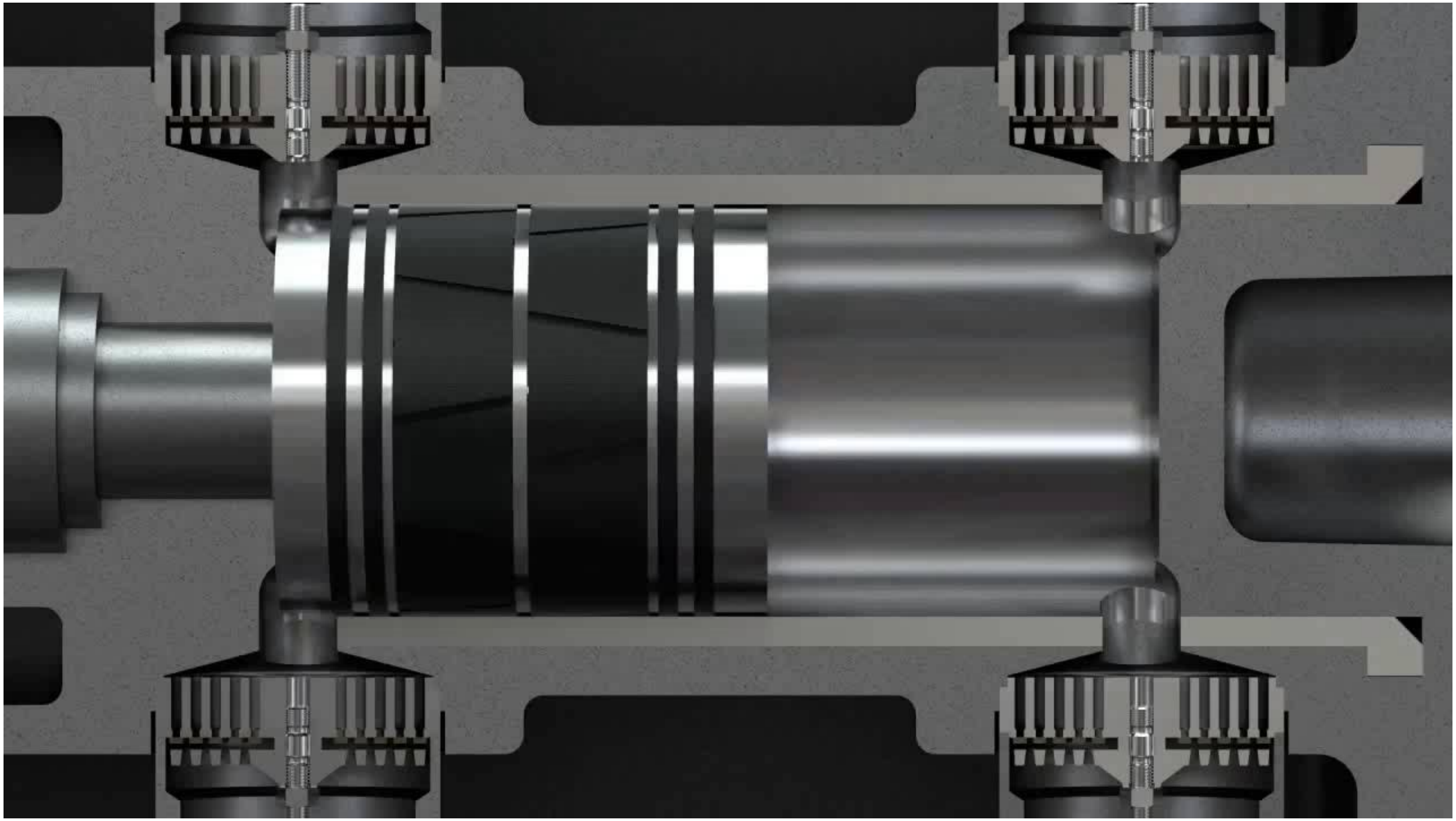
with power consumption P and
isobaric heat capacity c_p

Reversible adiabatic compression: $T_d = T_s * \left(\frac{p_d}{p_s}\right)^{\frac{\kappa-1}{\kappa}}$

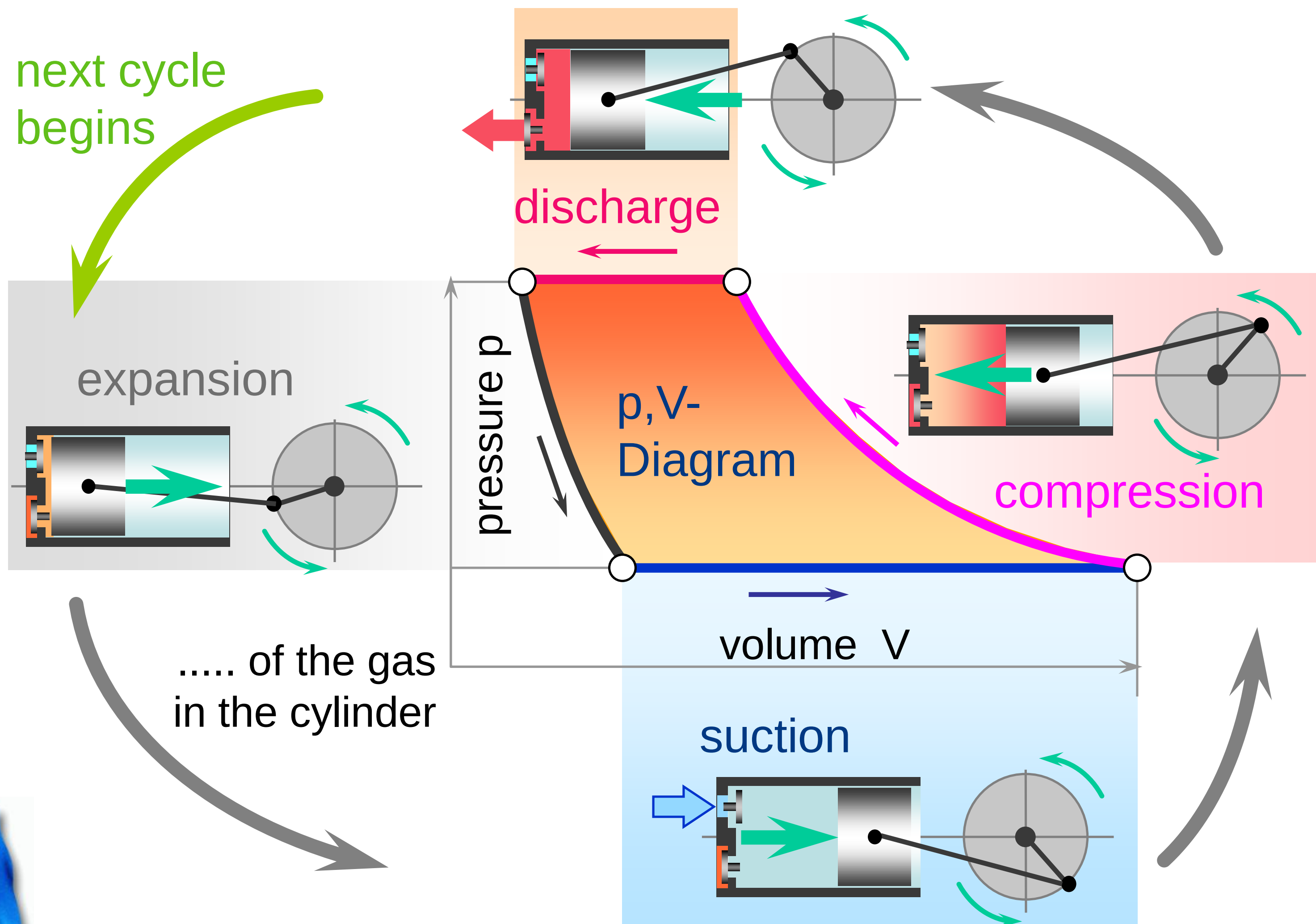
$$\Rightarrow P = \dot{m} * c_p * T_s * \left[\left(\frac{p_d}{p_s}\right)^{\frac{\kappa-1}{\kappa}} - 1\right]$$

Note: losses increase discharge temperature and thereby power consumption!

Compression Cycle – p-V-Diagram



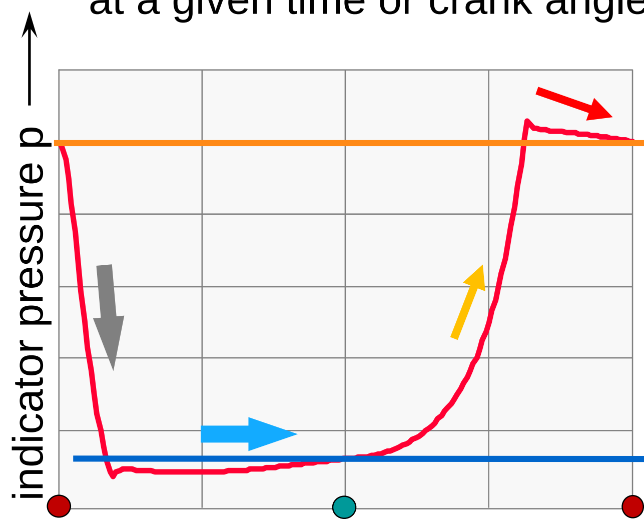
Compression Cycle – p-V-Diagram



Compression Cycle – p-V-Diagram

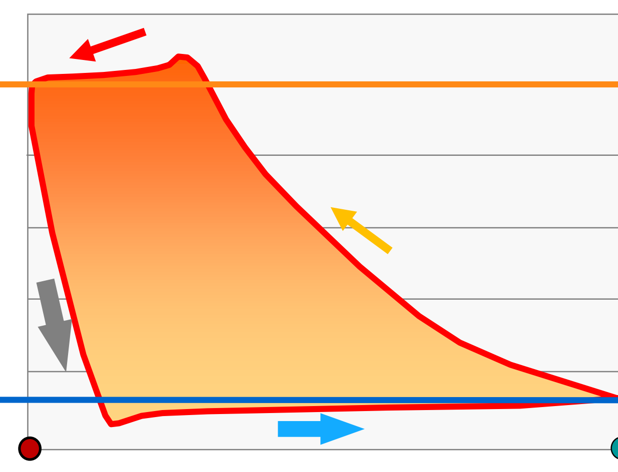
Pressure - Time Diagram p,t

shows the pressure in the cylinder at a given time or crank angle

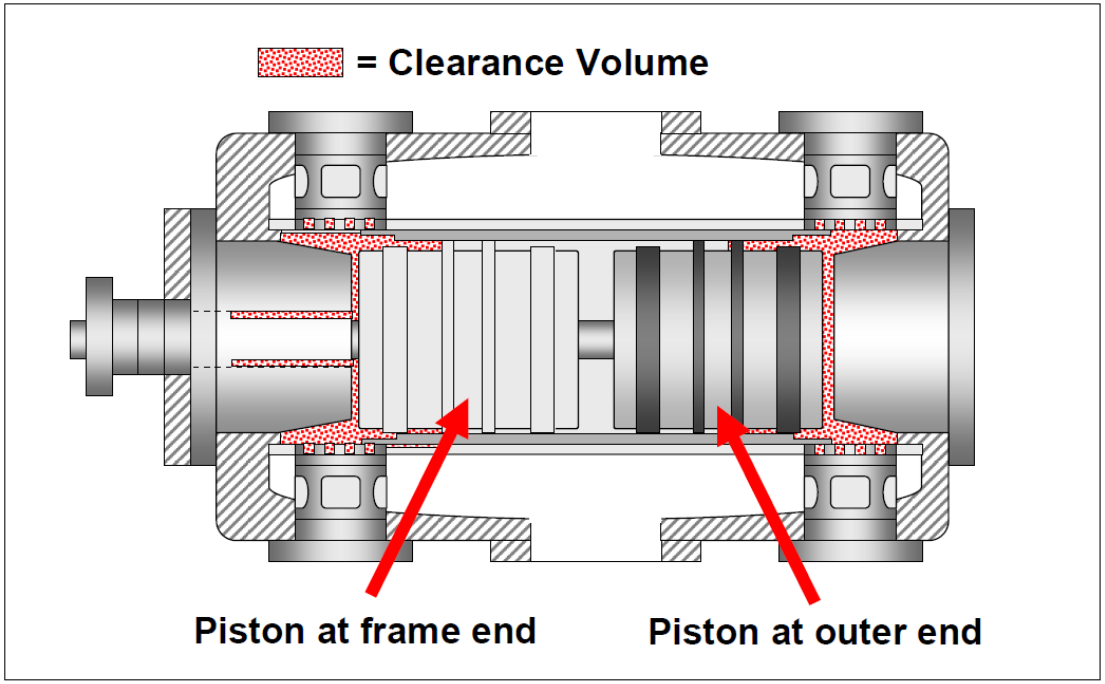


Pressure - Volume Diagram p,V

shows the pressure in the cylinder at a given volume or piston position



Compression Cycle – Clearance

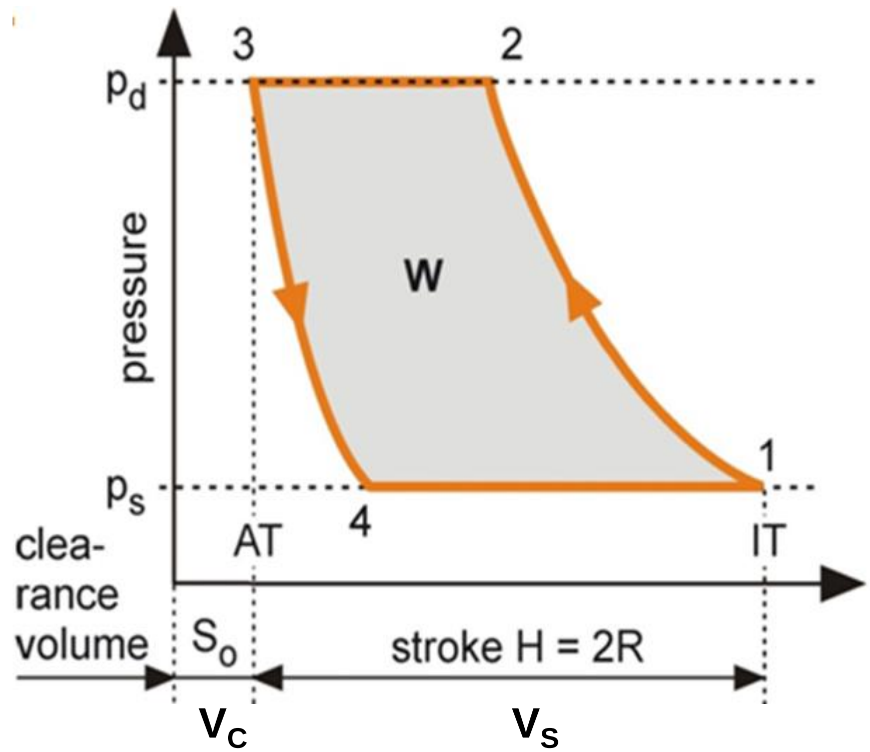


•Clearance refers to the volumes in each end of the cylinder that retain gas after the piston has stopped

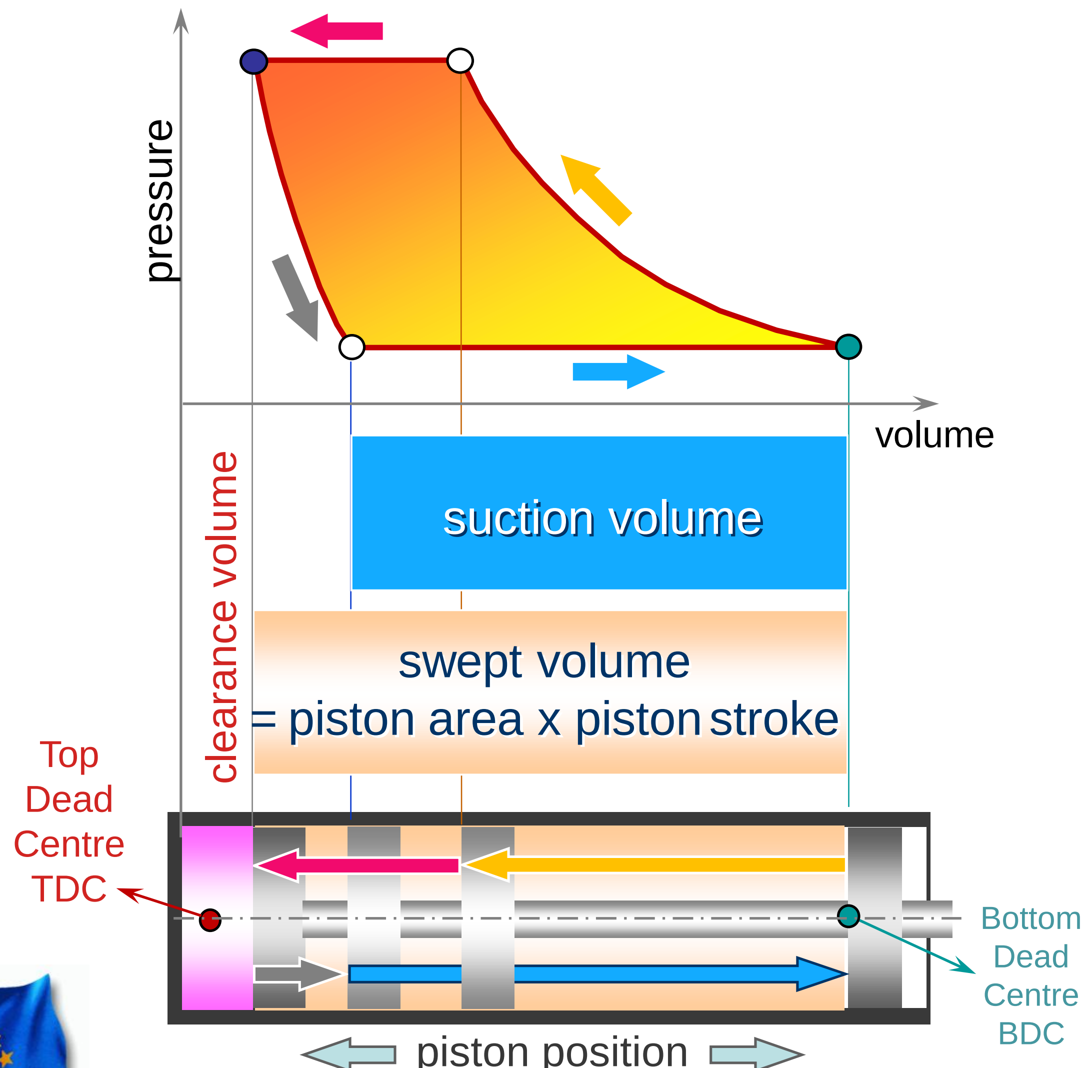
•Gas cannot be displaced; cylinder is not 100% efficient

Compression Cycle – Volumetric Efficiency

- **Volumetric efficiency**
- $\eta_{vol} = \frac{V_1 - V_4}{V_1 - V_3}$
- with $p_1 = p_4; p_2 = p_3;$
  $p_2 V_2^{\kappa} = p_3 V_3^{\kappa}$
- $\eta_{vol} = 1 - \frac{V_3}{V_1 - V_3}\left[\left(\frac{p_2}{p_1}\right)^{\frac{1}{\kappa}} - 1\right] =$
  $1 - \frac{V_C}{V_S}\left[\left(\frac{p_2}{p_1}\right)^{\frac{1}{\kappa}} - 1\right]$



Compression Cycle – Volumetric Efficiency



Volumetric efficiency

$$\eta_v = \frac{\text{suction volume}}{\text{swept volume}}$$

$$\lambda = 1 - \sigma_0 (\Pi^{\frac{1}{\kappa}} - 1) \boxed{- L}$$

λ filling efficiency= volumetric efficiency η_V minus losses L

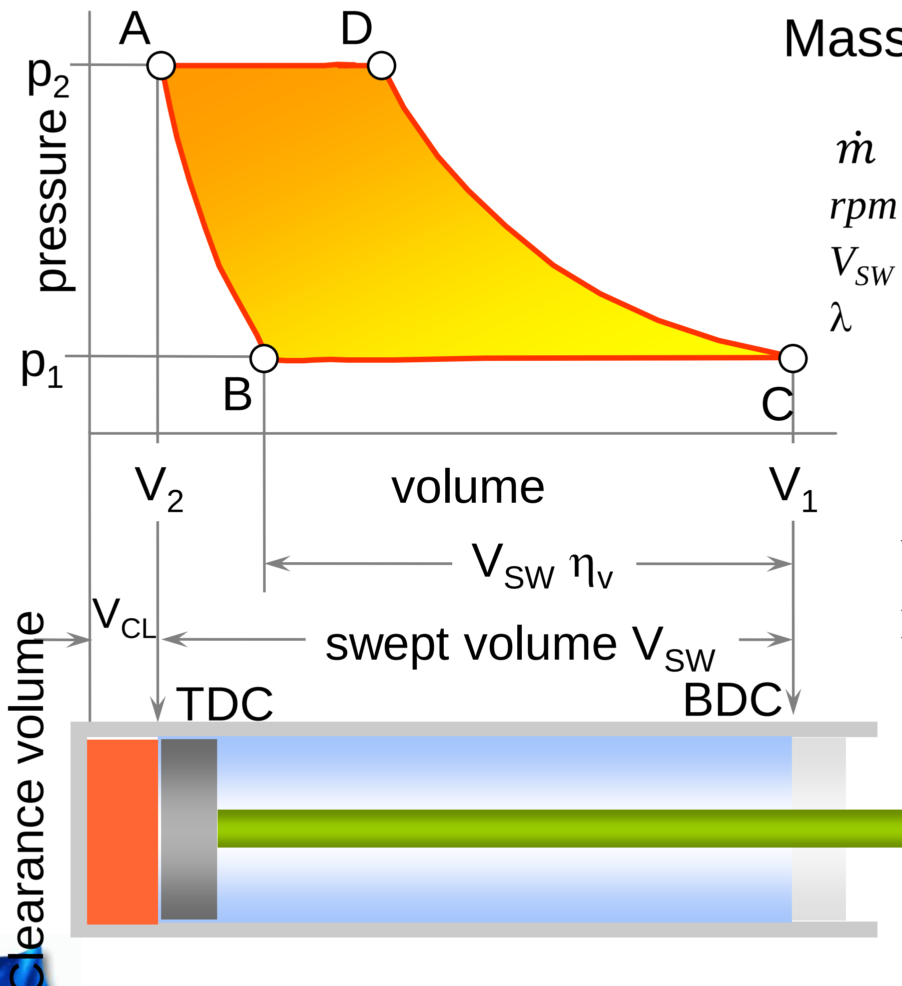
κ polytropic exponent

Π compression ratio p_2/p_1

σ_0 clearance volume referred to swept volume

L loss factor accounting for losses associated with leakage, heating up of gas during suction,….

Compression Cycle – Mass Flow



Mass flow $\dot{m} = \rho_s * V_{SW} * \lambda * rpm/60$

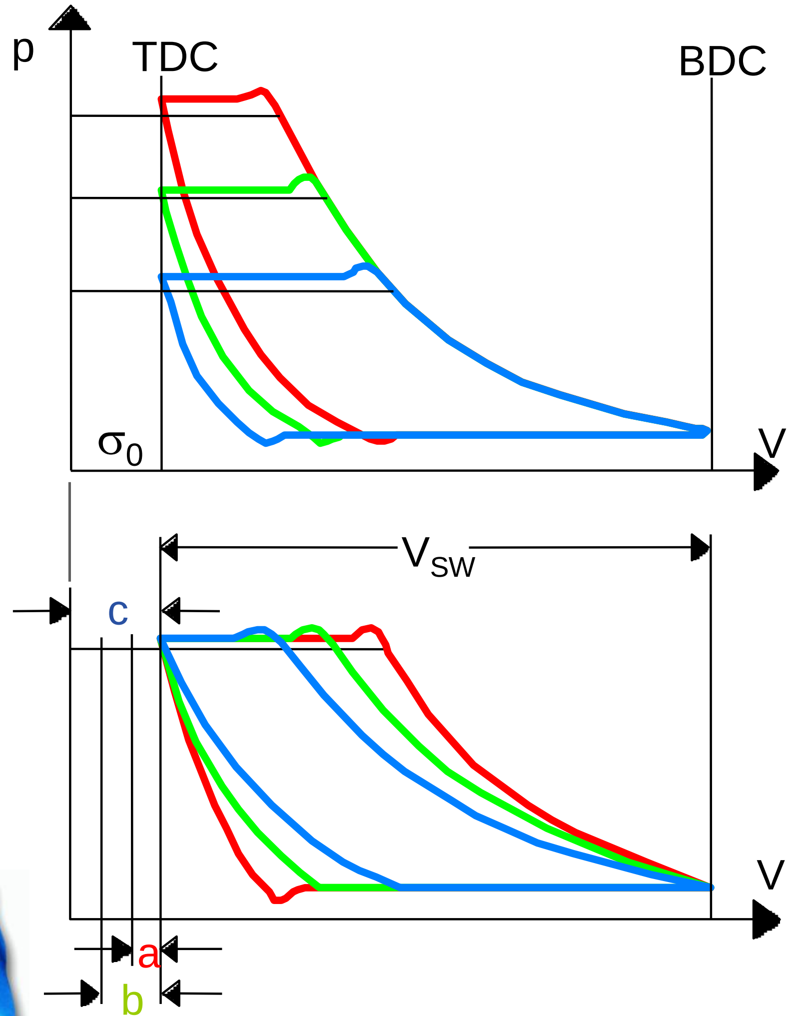
$\dot{m}$	adiabatic mass flow (kg/s)
rpm	speed (min^{-1})
V_{SW}	swept volume
λ	filling efficiency= volumetric efficiency η_V minus losses L

$$\lambda = 1 - \sigma_0 (\Pi^{\frac{1}{\kappa}} - 1) - L$$

κ	polytropic exponent
Π	compression ratio p_2/p_1
σ_0	clearance volume referred to swept volume
L	loss factor accounting for losses associated with leakage, heating up of gas during suction,….

Compression Cycle – Different p-V-Diagrams

Influence of pressure ratio p2/p1 and clearance volume σ_0



Influence of pressure ratio p2/p1 on volumetric efficiency:

4, 6, 8

clearance σ_0 constant 10%V_{SW}

Influence of clearance σ_0 on vol. efficiency:

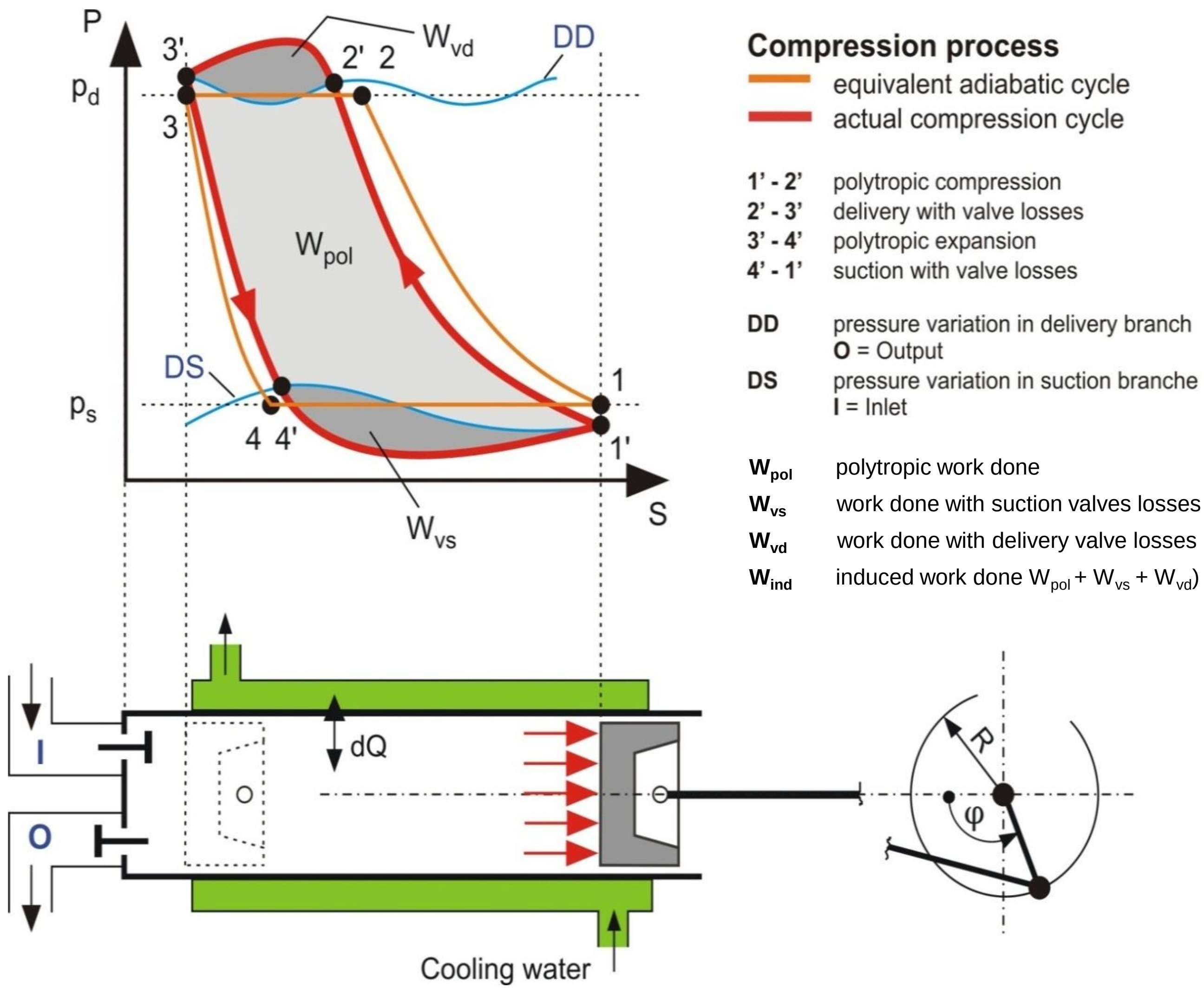
a: 6%,

b: 10%,

c: 15 % of V_{SW}

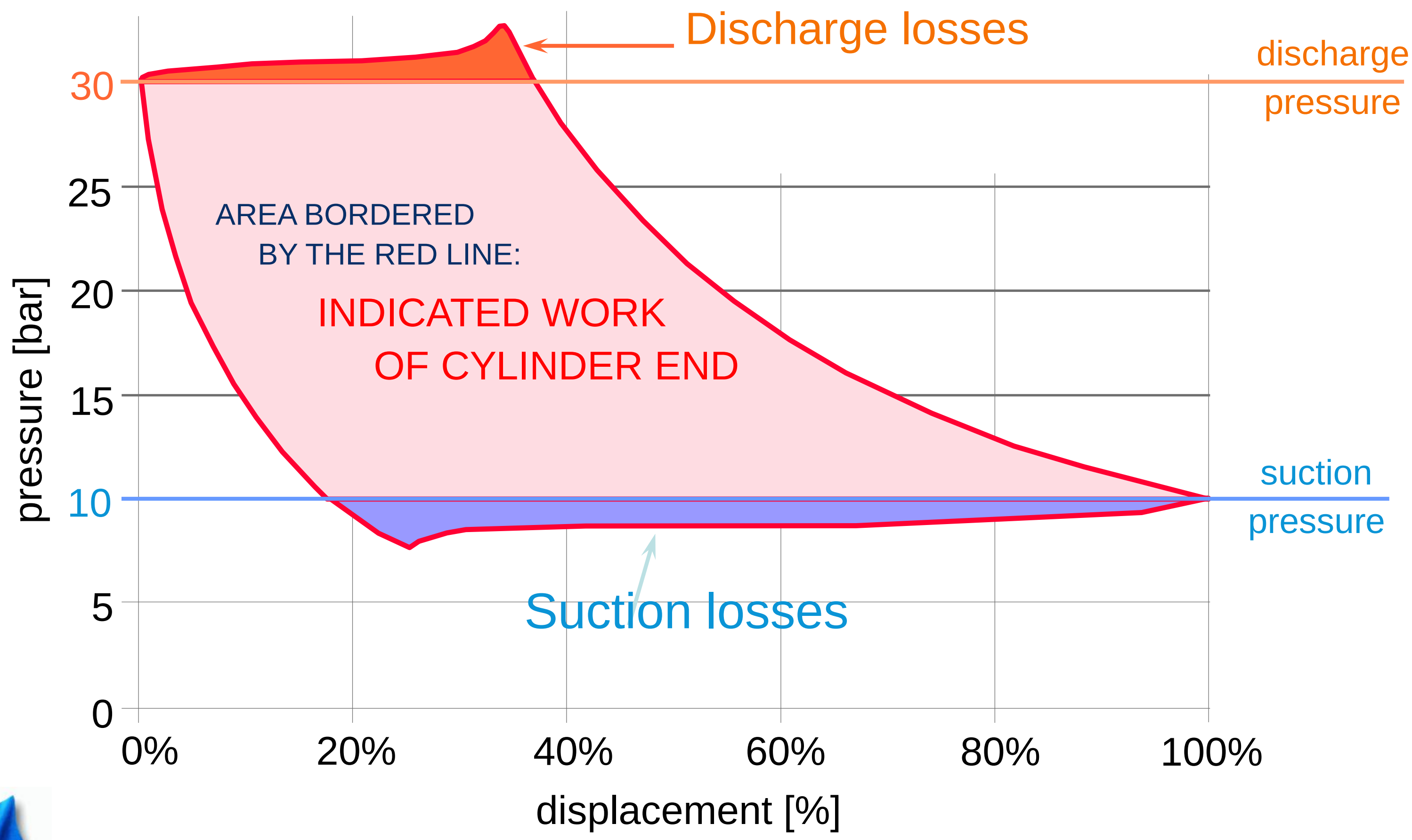
Pressure ratio p_2/p_1 constant

Compression Cycle – Actual p-V-Diagram



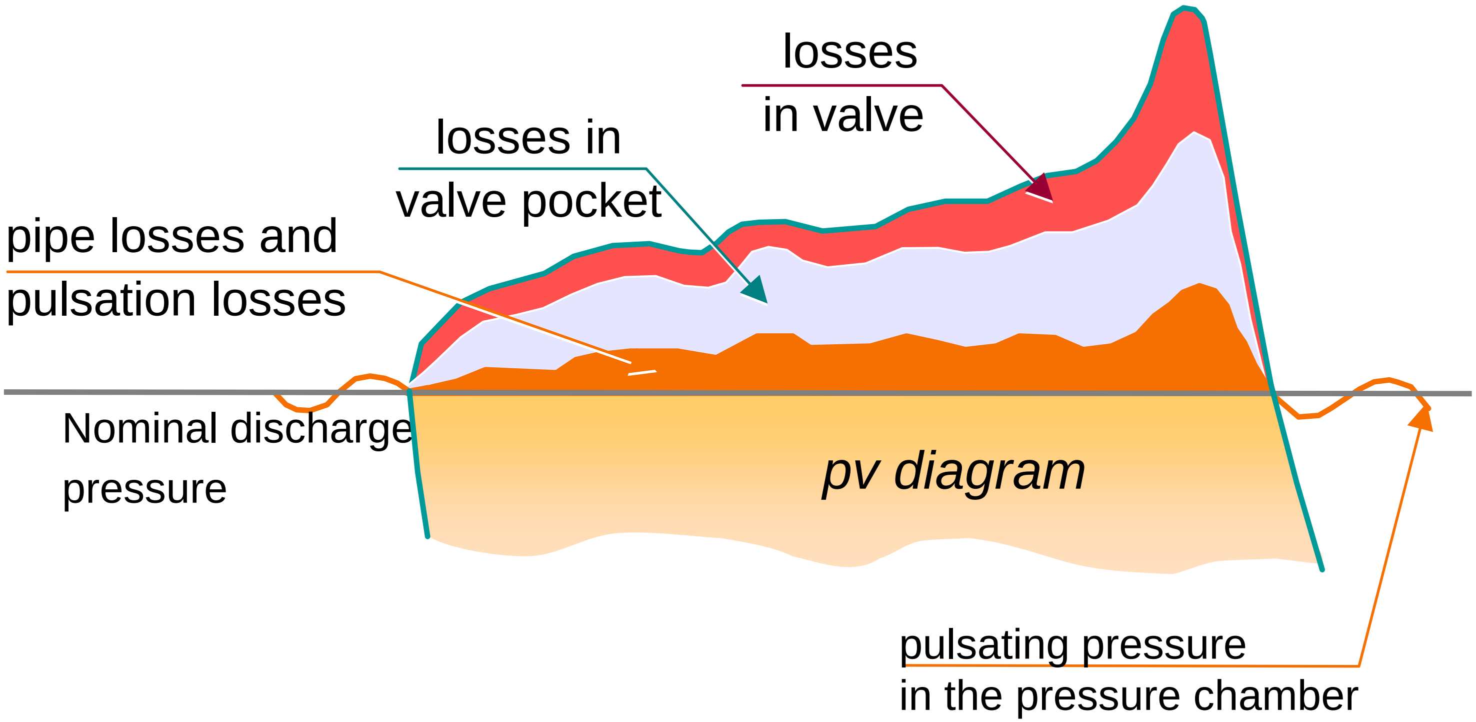
Compression Cycle – Losses

A certain % of compressor work is lost due to losses (suction and discharge

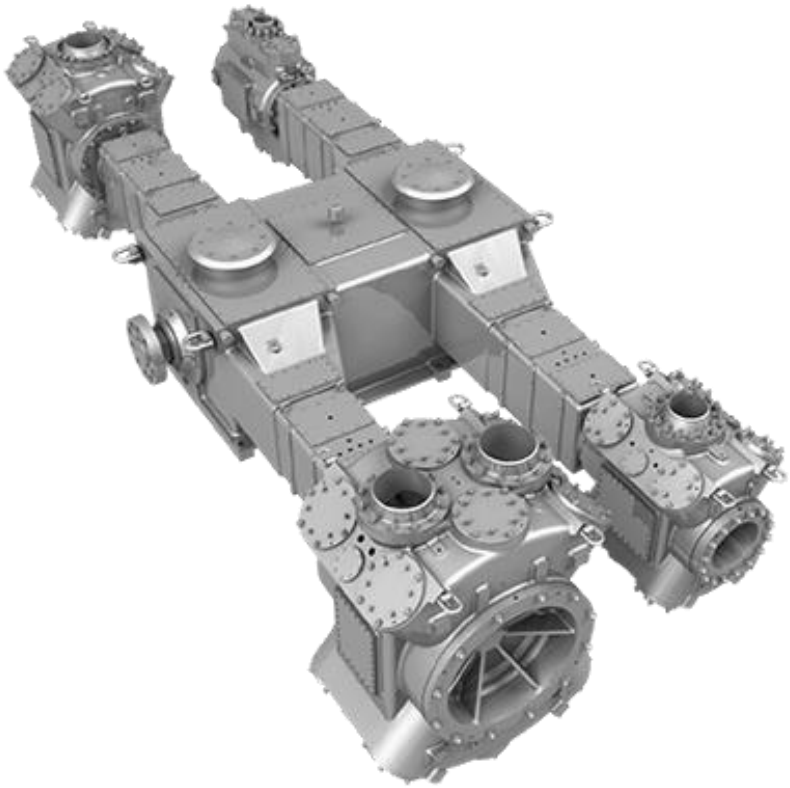


Compression Cycle – Losses

The areas exceeding nominal discharge pressure show the different losses at the delivery side.



Sizing – Multistage Compressors



Different reasons for multistage compressor:

1. Temperature – Compression ratio
2. Efficiency – Volume flow and power consumption
3. Load on compressor parts

Sizing – Multistage Compressor

Compression ratio - Temperature

The adiabatic compresson temperature

$$T_2 = T_1 \cdot \left(\frac{p_2}{p_1} \right)^{\frac{\kappa - 1}{\kappa}}$$

limits the pressure ratio in one stage as neither material nor gas may stand such a high discharge temperature value.

Practice shows, that reasonable maximal pressure ratios per stage are:

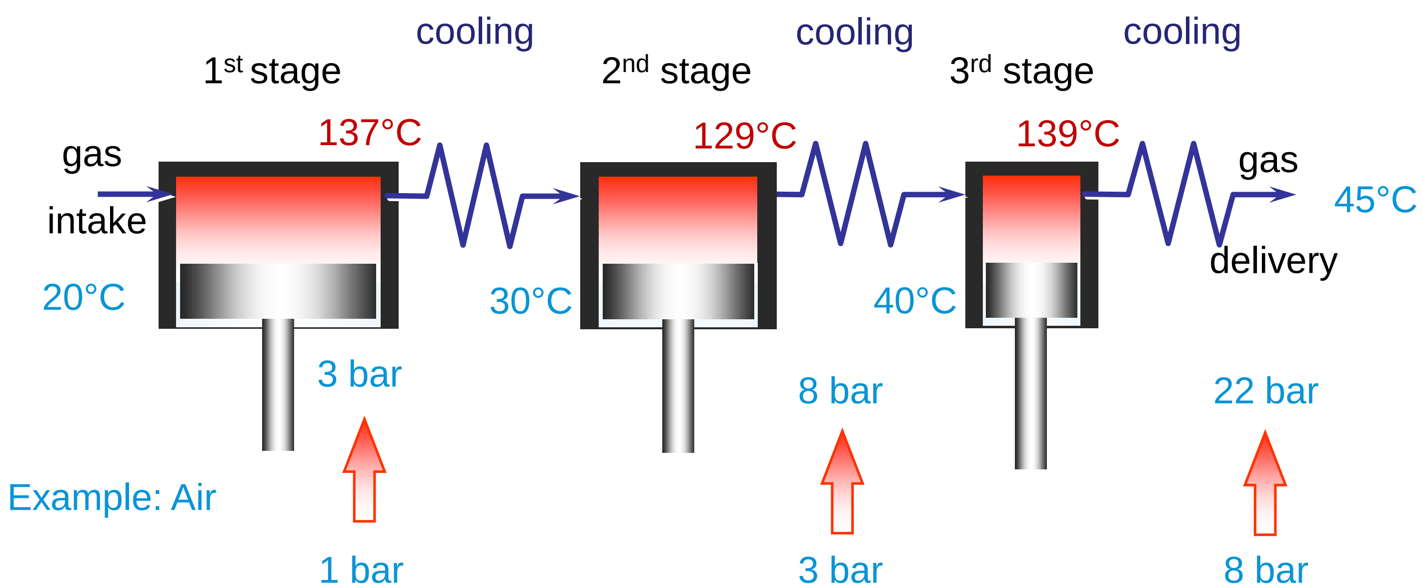
$p_2/p_1 = 5$ for polyatomic gases with $\kappa = 1{,}3$ (natural gas, CO_2 etc. …..)
$p_2/p_1 = 4$ for diatomic gases with $\kappa = 1{,}4$ (air, N_2, H_2, CO etc. …..)
$p_2/p_1 = 3$ for monoatomic gases $\kappa = 1{,}67$ (He, Ne, Xe, Ar etc. …..)

Multistage Compressors

Sizing – Multistage Compressors

Compression Ratio - Temperature

- Achieve higher pressures
- Needs to be cooled down between stages in order to avoid exceeding permissible temperature for compressor materials and lubricating oil.

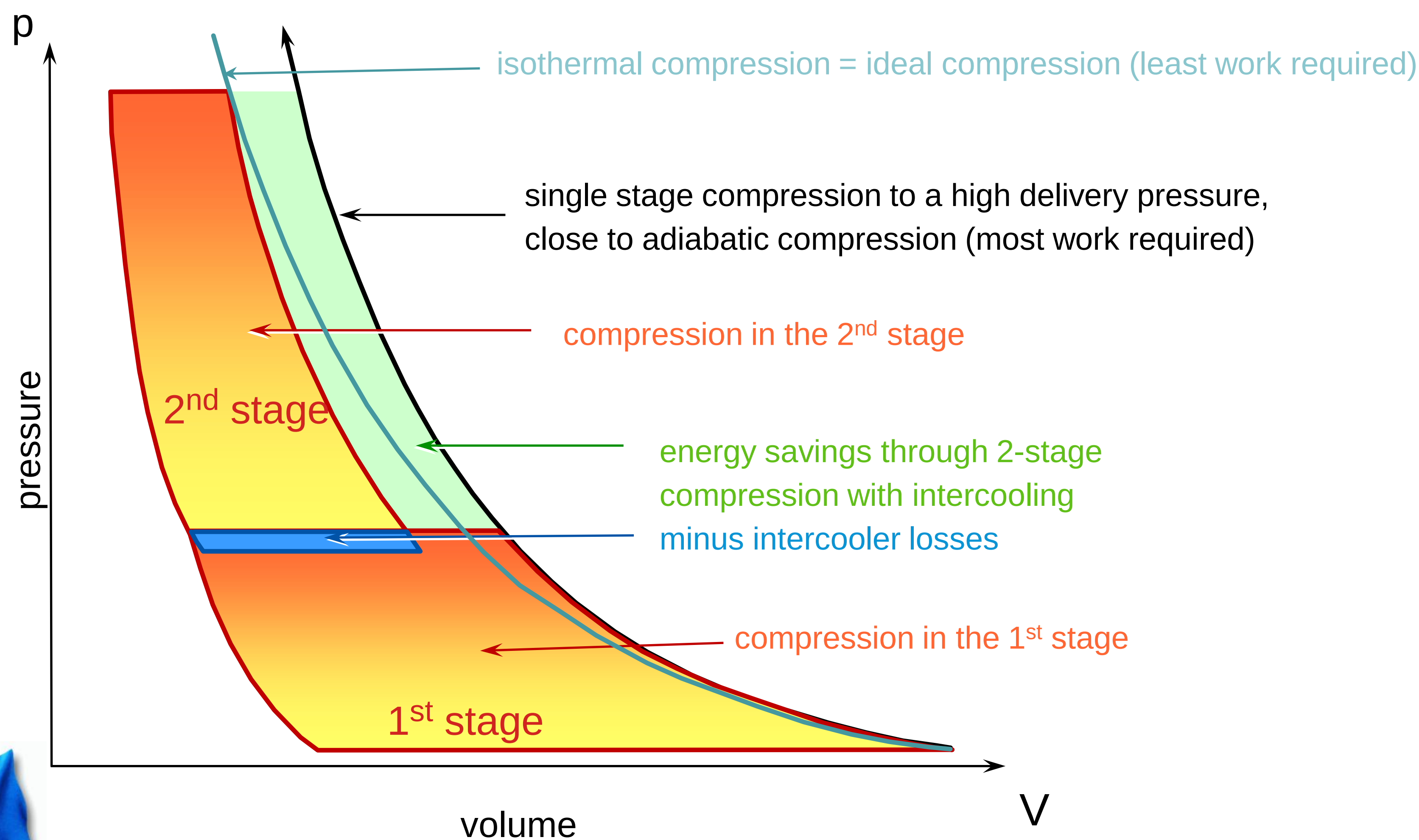


Discharge temperature: $$T_d = T_s * \left(\frac{p_d}{p_s}\right)^{\frac{\kappa-1}{\kappa}}$$

Pd … Discharge pressure
Ts … Suction temperature
PS … Suction pressure
k … Isentropic coeff.

Sizing – Multistage Compressors

Efficiency – Power Consumption



Sizing – Multistage Compressors

Example

Compression of Ethene from 1 to 8 bara, suction temperature 20°C (each stage), κ=1.24, clearance volume of each stage: 10% of swept volume, interstage pressure: 2.83 bara

Single-stage compressor	Multi-stage compressor
$T_2 = T_1 \cdot \left(\frac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}} = 438°\mathrm{K} = 165°\mathrm{C}$	$T_2 = 273 \cdot \left(\frac{3.16}{1}\right)^{\frac{1.24-1}{1.24}} = 358°\mathrm{K}$ $= 85°\mathrm{C}$
$\eta_{vol} = 1 - \frac{V_C}{V_S}\left[\left(\frac{p_2}{p_1}\right)^{\frac{1}{\kappa}} - 1\right] = 0.565$	$\eta_{vol} = 1 - 0.1\left[\left(\frac{2.828}{1}\right)^{\frac{1}{1.24}} - 1\right]$ $= 0.869$
	Power consumption is 9% lower

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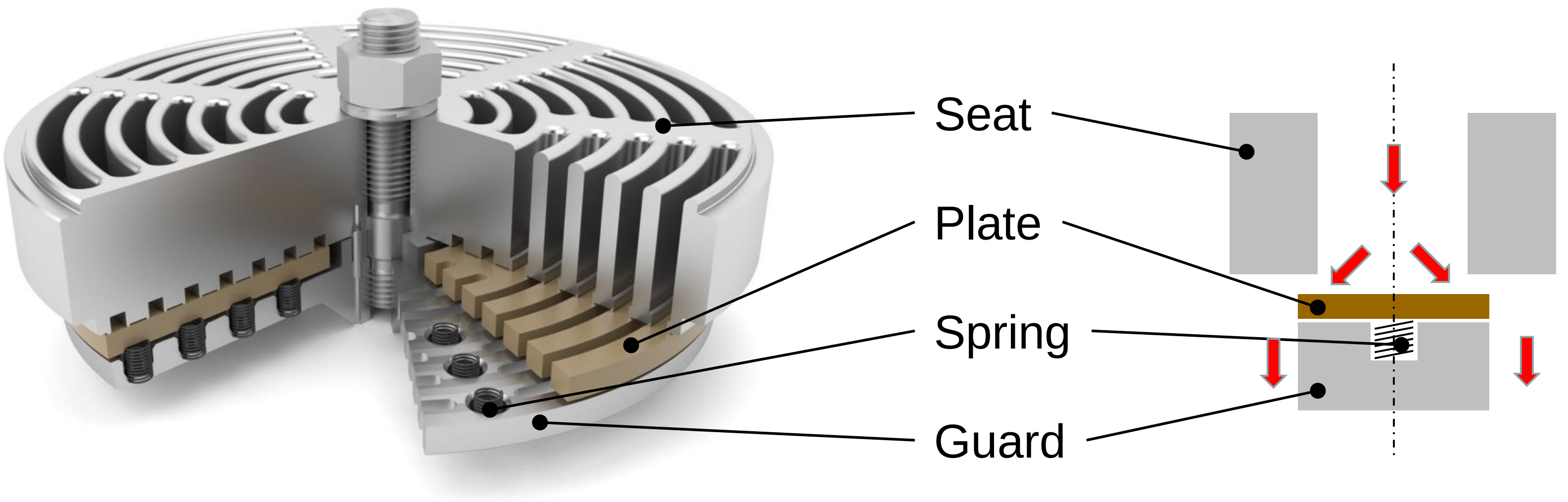
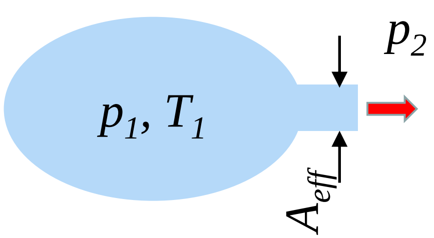
Sizing – Valve Model



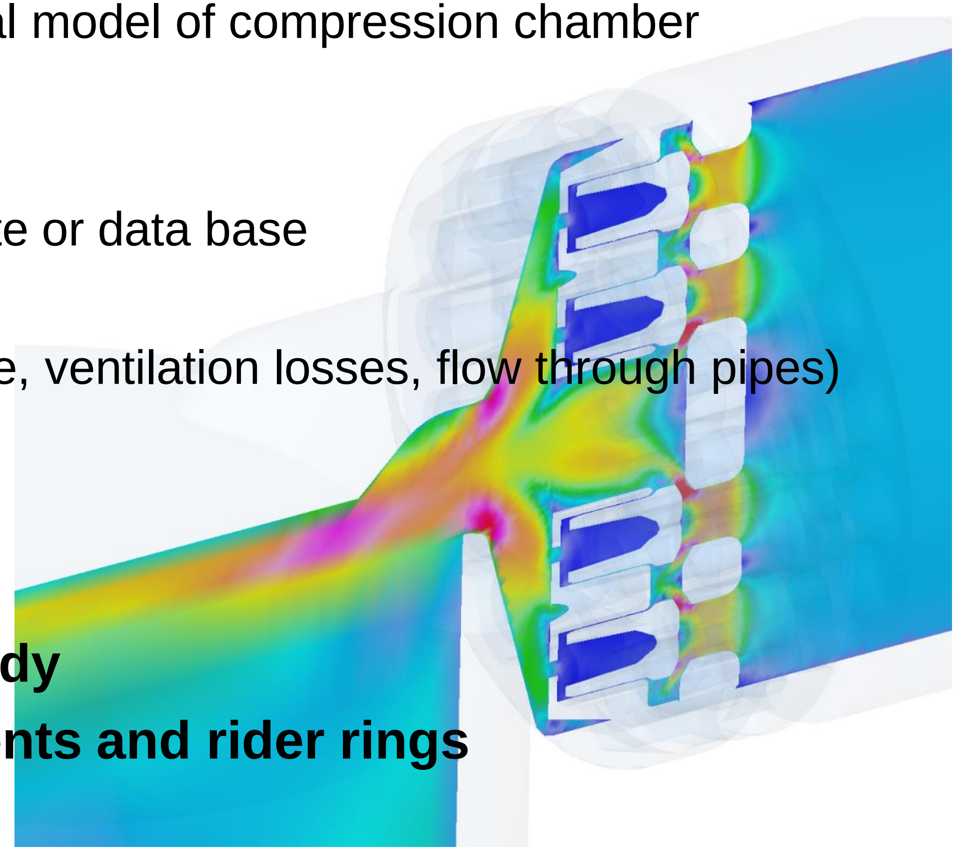
Flow through valve = Isentropic flow through a nozzle



Effective Flow Area A_{eff} must be determined using CFD or experiments

Sizing – Method

- **Thermodynamic System - Numerical Simulation of Compression Chamber:**
  - 0-, 1-, 2- or 3 dimensional model of compression chamber (conservation laws)
  - Valve model
  - Real gas equation of state or data base
  - Heat transfer
  - Model for losses (leakage, ventilation losses, flow through pipes)
  - Motion of drive chain
- **Mechanical Strength**
- **Dynamics of drive chain**
- **Vibration and pulsation study**
- **Tribology for sealing elements and rider rings**



Summary

- A reciprocating compressor is the most efficient device to compress gas
- Although seemingly simple, a lot of fluid mechanics and thermodynamic knowledge is required
- For high pressures and low to high volume flows the reciprocating compressor is the best choice
- There is no substitute for the reciprocating compressor